Preface: Math Tools: A Cure for the Common Core

Almost a decade ago, we embarked on a journey guided by a simple question: *What would a tools-based approach to mathematics instruction look like?* What we wanted to develop was a new kind of resource—a practical, easy-to-use collection of "math tools" that would respond directly to the different instructional challenges teachers of mathematics face. This journey culminated in *Math Tools, Grades 3–12: 64 Ways to Differentiate Instruction and Increase Student Engagement* (Silver, Brunsting, & Walsh, 2008).

Let's fast forward a few years. In light of new experiences, new perspectives (welcome, Ed Thomas), and new challenges facing mathematics educators, we felt the time was right for an enhanced and updated edition of *Math Tools.* Of course, not all the challenges teachers of mathematics confront are new—the tools in this enhanced edition of *Math Tools* remain excellent techniques for helping teachers increase student engagement, differentiate instruction, and design comprehensive lessons and units. We've also taken care to highlight these tools as effective options for formatively assessing student progress and integrating technology and multimedia into the classroom. But, as the subtitle to this preface reminds us, the emergence of the Common Core State Standards for Mathematics (National Governors Assocation Center for Best Practices [NGA], Council of Chief State School Officers [CCSSO], 2010b) is the most significant challenge facing teachers of mathematics. Thus, the emergence of the Common Core documents—and their expectations—have guided much of our work in developing this new and revised version of *Math Tools.*

The Common Core State Standards for Mathematics are new, but not radical. They bring together Standards for Mathematical Content (specific for kindergarten through high school) and Standards for Mathematical Practice, which "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (NGA Center, CCSSO, 2010b, p. 6). These standards are built from decades of collaboration and progress in developing a more rigorous mathematics curriculum. The goal of the Common Core is to "describe a coherent, focused curriculum that has realistically high expectations and supports an equitable mathematics education for all students" (2010) is supported by the National Council of Teachers of Mathematics (NCTM), the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), and the Association of Mathematics Teacher Educators (AMTE).

The longstanding positions and principles of these organizations are reflected in the central elements of the Common Core State Standards for Mathematics, including the notion that "all students need to develop mathematical practices such as solving problems, making connections, understanding multiple representations of mathematical ideas, communicating their thought processes, and justifying their reasoning" (NCTM, 2010).

And, as John Kendall (2011) notes in *Understanding Common Core State Standards,* "[T]he standards' emphasis on conceptual understanding is evident not just in its Standards for Mathematical Content but also in its Standards for Mathematical Practice, which show indebtedness to NCTM's (2000) *Principles and Standards for School Mathematics* and to the strands of mathematical proficiency identified in the National Research Council's influential text *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001)" (p. 23).

In the second edition of *Math Tools,* we do not attempt to provide a primer on the Common Core State Standards for Mathematics. This work is done much better by other resources, such as NCTM's *Making It Happen: A Guide to Interpreting and Implementing Common Core State Standards for Mathematics* (2012). And, if you're looking for a general overview of the Common Core, we recommend the aforementioned *Understanding Common Core State Standards* (Kendall, 2011). But, if you're looking for a set of ready-to-use instructional tools that build the type of mathematical thinking and reasoning highlighted in the Standards for Mathematical Practice, then this new edition of *Math Tools* is your book.

Over the subsequent pages, you'll notice that we have put our focus on the Standards for Mathematical Practice. We've done so because they tell us how "developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years" (NGA Center, CCSSO, 2010b, p. 8). In other words, "These standards establish critical practices that are valued throughout the grade levels" (Kendall, 2011, p. 24). The practices serve as the "thinking glue" that holds the entire mathematics curriculum together, from kindergarten through high school.

Here are the eight Standards for Mathematical Practice (NGA Center, CCSSO, 2010b):

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

For students to consistently demonstrate these practices, teachers must routinely provide activities and lessons that support the development of requisite knowledge (e.g., vocabulary) and appropriate habits (e.g., justification of work).

We developed a new edition of *Math Tools* to help teachers in elementary, middle, and high school mathematics classrooms identify and select the best instructional tools to address the Standards for Mathematical Practice. The tools in this book support the development of students' reasoning and skills, and provide formative assessment opportunities that allow teacher and student to assess and refine students' thinking and learning. That's why we believe this enhanced edition of *Math Tools* can be the "cure" for the Common Core, helping to alleviate teachers' anxiety over implementing these standards by breaking down the Standards for Mathematical Practice and providing practical, effective, and easy-to-use instructional tools that address them.

Each math tool includes a "Building Common Core Thinking" section, which identifies the Standards for Mathematical Practice the tool supports by number, key word, and key phrase extracted from the practice description (see Figure B on pp. xiv–xv for a sampling of these key phrases). In addition, some math tools also include a Common Core "ELA Note." This note highlights any key College and Career Readiness Anchor Standards for Reading, Writing, Speaking and Listening, or Language (NGA Center, CCSSO, 2010a) that the tool can support. For example, the Building Common Core Thinking section for the Explaining Solutions tool (p. 159) is shown in Figure A below:

Figure A Building Common Core Thinking Section from Explaining Solutions Tool

Building Common Core Thinking

Explaining Solutions builds students' reflecting, reasoning, and sense-making habits as they explain and communicate the problem-solving process behind a solution. Explaining Solutions supports the following Standards for Mathematical Practice (MP):

- ***(MP 1) Sense:*** explaining correspondences between equations, descriptions, tables, and graphs or diagrams
- ***(MP 2) Reason:*** *learning to create coherent representations*
- ***(MP 3) Argument:*** supporting and justifying conclusions
- ***(MP 4) Model:*** interpreting mathematical work in the light of a context
- ***(MP 6) Precision:*** formulating careful explanations

ELA Note: Explaining Solutions can also help support the Common Core Anchor Standards for Writing and Language related to informative/explanatory writing (W.CCR.2), on-task writing (W.CCR.4), conventions of standard written English (L.CCR.1, L.CCR.2), and use of vocabulary (L.CCR.6).

It is our sincere hope that teachers of mathematics will use this new edition of *Math Tools* as a rich and practical resource for developing "mathematically proficient students," thus preparing students for success in college and careers of the 21st century and "curing" the anxiety building in many schools surrounding the Common Core State Standards for Mathematics.

Figure B Standards for Mathematical Practice—Key Words and Phrases

Standard for Mathematical Practice (MP)	Key Word	Key Phrases
Mathematically proficient students . . . 1. Make sense of problems and persevere in solving them	Sense (MP 1)	• Explain to themselves the meaning of a problem • Look for entry points to a problem • Make conjectures • Plan a solution pathway • Consider analogous problems • Monitor and evaluate their progress • Transform expressions, change viewing windows, etc. • Explain correspondences between equations, verbal descriptions . . . • Use concrete objects or pictures to help conceptualize • Check answers to problems using a different method • Ask themselves, "Does this make sense?" • Understand others' approaches to solving complex problems
Mathematically proficient students . . . 2. Reason abstractly and quantitatively	Reason (MP 2)	• Make sense of quantities and their relationships • Decontextualize problems • Represent problems symbolically, and manipulate those symbols as needed • Contextualize problems • Pause to look deeper into symbols • Create a coherent representation of a problem • Make meaning of quantities, not just compute them • Flexibly use different properties of operations and objects
Mathematically proficient students . . . 3. Construct viable arguments and critique the reasoning of others	Argument (MP 3)	• Understand and use assumptions, definitions, and previously learned information • Make conjectures and build a logical progression of statements to explore the truth of their conjectures • Analyze situations by breaking into cases • Recognize and use counterexamples • Justify and communicate their conclusions and respond to the arguments of others • Reason inductively • Compare plausible arguments, distinguish correct logic, and explain incorrectness • Construct arguments using concrete referents: objects, drawings, diagrams, actions • Determine domains to which an argument applies • Listen to and read arguments of others, decide if they make sense • Ask useful questions to clarify and improve arguments
Mathematically proficient students . . . 4. Model with mathematics	Model (MP 4)	• Apply the mathematics they already know to solve real, everyday problems • Write an equation or inequality to describe a situation • Apply reasoning to analyze a problem • Solve or design a problem (e.g., use a function to describe how one quantity depends on another) • Make valid approximations and assumptions to simplify a situation • Identify need for revisions • Identify important quantities, map relationships using a variety of techniques/draw conclusions • Interpret and reflect on results

Standard for Mathematical Practice (MP)	Key Word	Key Phrases
Mathematically proficient students . . . 5. Use appropriate tools strategically	Tools (MP 5)	• Develop proficiency using a variety of tools: pencil and paper; calculator and software; ruler, compass, and protractor • Make use of available and appropriate tools, recognizing their value and limitations • Use tools to explore and deepen understanding by visualizing the problem in different ways and/or comparing possible solutions with the data • Use technology to explore problems that they could not study using pencil and paper alone—for example, calculators to compute problems with much data or very large numbers; graphing calculators to preview concepts traditionally reserved for later math classes; computers to analyze complex problems that could not otherwise be studied; or the Internet to research or explore mathematical content
Mathematically proficient students . . . 6. Attend to precision	Precision (MP 6)	• Communicate precisely both verbally and in writing • Use complete sentences for both written and verbal answers • Use clear, accurate definitions • State meaning of mathematical symbols • Label details accurately (e.g., units, axes, tables and graphs) • Calculate accurately and efficiently, expressing answers with appropriate precision and units • Give carefully formulated explanations to others • Examine solutions or claims in light of definitions and constraints
Mathematically proficient students . . . 7. Look for and make use of structure	Structure (MP 7)	• Look closely to discern a pattern or structure • See parts in relationship to the whole • Consider alternative perspectives for a given problem or situation • Break complicated problems into smaller, simpler parts • Recognize significance or underlying structure of a problem
Mathematically proficient students . . . 8. Look for and express regularity in repeated reasoning	Repetition (MP 8)	• Notice patterns in calculations or solutions to problems • Generalize methods and shortcuts • Maintain oversight of the process • Attend to the details • Continually check for reasonableness

Contents

Additional materials and resources related to *Math Tools, Grades 3–12: 60+ Ways to Build Mathematical Practices, Differentiate Instruction, and Increase Student Engagement*, Second Edition, can be found at http://www.corwin.com/mathtools3-12.

MATHEMATICAL CONVENTIONS

Purpose

The world of mathematics is filled with procedures, notations, and common practices that, while seemingly insignificant to many students, help to clarify abstract concepts and make calculations manageable. We call these common protocols conventions. A *mathematical convention* is a

- General agreement or acceptance of certain practices or attitudes.
  - By convention, on a graph the axes are labeled.
  - Polynomials are written with terms in decreasing order.
  - There are conventions for writing restrictions when theorems, problems, or properties include variables in the denominators of fractions.
- Procedure or technique widely accepted by a group; a custom.
  - Restrictions associated with answers are stated explicitly if not clearly understood.
  - For example, in some classrooms, the teacher requires three points to be graphed on a line instead of two points.

Students need to make decisions about when conventions are being used appropriately and explain why they made their choices. In this way, the Mathematical Conventions technique helps students to see mathematics terms and notations not just as jargon or glyphs, but also as helpful and useful expressions of mathematical concepts.

Overview

The Mathematical Conventions technique enables students to increase their fluency in the language of mathematics. To use the Mathematical Conventions technique, the teacher identifies the conventions associated with a lesson or unit. These conventions can be words, rules, symbols, notations, or protocols for formatting or simplifying expressions.

Built into the Mathematical Conventions technique is an assessment process in which students examine two options (one that uses the convention correctly and one that does not). Students select the accepted, or more correct, convention and then justify their choices by writing a brief explanation.

Building Common Core Thinking

The Mathematical Conventions tool enhances students' quantitative reasoning, focuses student attention on parameters and constraints, and builds their overall fluency with speaking, reading, and writing the language of math. Mathematical Conventions supports the following Standards for Mathematical Practice (MP):

- ***(MP 1) Sense:*** analyzing and applying constraints
- ***(MP 2) Reason:*** working with quantitative symbolic expressions and knowing and using different properties of operations

- ***(MP 6) Precision:*** calculating efficiently and communicating appropriately
- ***(MP 8) Repetition:*** noticing regularity in symbol manipulation and attending to details

ELA Note: Depending on the activity, Mathematical Conventions can also complement the development of Common Core Anchor Standards for Language related to the conventions of standard English (L.CCR.1, L.CCR.2).

Steps

1. Identify all of the relevant conventions that will be used for a lesson or unit and explain the rationale for each convention.
2. Allow students time to briefly review the conventions.
3. Provide students with a worksheet that has accepted conventions paired up with common alternatives.
4. Have students choose which item in each pair is the preferred, or more correct, use of the convention. Students then explain in writing the thinking behind their choices.

Examples

In the examples of Mathematical Conventions shown in Figures 1.13 and 1.14, students are asked to pick between two options and to explain their choices in the spaces provided.

Figure 1.13 Mathematical Conventions for *Finding the Area of Geometric Figures*

	Option I	Option II	Explanation
A	Triangle $A = \frac{b \bullet h}{2}$	Triangle $A = \frac{1}{2}bh$	
B	Circle $A = \pi r^2$	Circle $A = r \bullet r \bullet 3.14159$	
C	Trapezoid $A = \frac{1}{2}h(b_1 + b_2)$	Trapezoid $A = h \bullet \frac{b_1 + b_2}{2}$	
D	Square $A = b \bullet h$	Square $A = s^2$	

Figure 1.14 Mathematical Conventions for *Solving Equations and Inequalities*

	Option I	Option II	Explanation
A	$-\frac{-2}{-3}=\frac{2}{-3}$	$-\frac{-2}{-3}=\frac{-2}{3}$	
B	$\frac{1}{x-1}-\frac{x}{x-1}=\frac{1-x}{x-1}$ $=-\frac{x-1}{x-1}$ $=-1$	$\frac{1}{x-1}-\frac{x}{x-1}=\frac{1-x}{x-1}$ $=-1,\quad x\neq 1$	
C	$-2x+8>4$ $\frac{-2x+8}{2}>\frac{4}{2}$ $-x+4>2$ $-x>-2$ $(-1)(-x)<(-1)(-2)$ $x<2$	$-2x+8>4$ $-2x+8-8>4-8$ $-2x>-4$ $\frac{-2x}{-2}>\frac{-4}{-2}$ $x<2$	

THINKING NOTES

Purpose

Making meaningful notes is a crucial skill for students to develop, but teachers often do not encourage students to make notes or give them ample opportunity to develop as "notemakers." Significant practice time is given to the manipulation of numbers and symbols, as preparation for future problem solving, but little time is afforded to students to make notes about their underlying thinking and reasoning. Thinking Notes places the mechanics of problem solving within a notemaking framework, enabling students to deepen their understanding of important mathematical processes while they develop the skill of making notes.

Overview

For students to produce quality notes, they must elaborate on their work and expose their thinking. Unlike students who passively *take notes* by simply copying the teacher's words and work, students who make notes are actively thinking—they're analyzing the steps of a solution, describing and explaining their work at each step, and capturing questions that arise while they work. As they process steps of the solution, their reasoning is strengthened. The Thinking Notes framework increases student ownership of their learning—at each step in the problem-solving process, students ask and answer the question, "Why did I (take a particular step)?" and thereby focus and deepen their understanding, recalling prior knowledge, patterns, and the goals of distinct steps. They also communicate their reasoning or, if unable to clearly justify why they took a step, they record a question about that step in the process. This step-by-step approach to Thinking Notes works well with the sequential nature of many mathematical processes.

Building Common Core Thinking

Thinking Notes requires students to be deliberate in their problem solving *and* thinking, deepening their understanding while building their confidence for reasoning through problems. Thinking Notes supports the following Standards for Mathematical Practice (MP):

- ***(MP 1) Sense:*** analyzing and explaining transformations of equations
- ***(MP 2) Reason:*** decontextualizing and manipulating symbols and using properties
- ***(MP 3) Argument:*** building logical progressions of statements
- ***(MP 6) Precision:*** clarifying reasoning and formulating explanations
- ***(MP 7) Structure:*** looking closely to discern patterns, stepping back for an overview
- ***(MP 8) Repetition:*** noticing repetitions in computations and similarity in methods

Steps

1. Write out the problem-solving steps for an equation (expression) in the center column of the three-column Thinking Notes form (see Figure 2.13).
2. Model (or review) the Thinking Notes with a sample problem. (*Note:* Until students are very familiar with the Thinking Notes form and process, you should model with a sample problem at least twice. For example, solve an inequality after an equation; perform one of the four operations with fractions, then with mixed numbers.)
3. Once students are comfortable with Thinking Notes, assign a problem for them to solve. (*Note:* Depending on the grade and content, you may want to have students work in small teams to solve a problem initially.)
4. Have students consider each step in the solution and complete the left and right columns of the Thinking Notes form. (*Note:* Encourage students to know and use the names of well-defined properties when writing.)

Examples

Figure 2.13 Student's Completed Thinking Notes Form for *Solving an Inequality*

What Did I Do?	Finding the Solution(s)	Explain Reason(s) Why (or Ask a Question)
Copied original problem	$32 + 3(2x - 12) > 5 - (4 + 9x)$	Starting point of work
Distributed the 3 times (2x – 12) and distributed the minus sign (-1) over (4 + 9x)	$32 + 6x - 36 > 5 - 4 - 9x$	Began simplifying both sides of equation by dealing with "( )" order of operations
Added like terms on both sides	$-4 + 6x > 1 - 9x$	Adding like terms simplifies each side
Add 9x to each side of the problem	$-4 + 15x > 1$	Addition Property of Equality results in having a single x term in equation
Add 4 to each side of the problem	$15x > 5$	Addition Property of Equality isolates the x term on one side
Divide each side by 15	$x > \frac{1}{3}$	Multiplication Property of Inequality helps solve for x

Visit the companion website for a blank "Thinking Notes Organizer."

The following examples show only the solution steps needed for Thinking Notes.

Dividing Mixed Numbers

What Did I Do?	Finding the Solution(s)	Explain Reason(s) Why (or Ask a Question)
	$7\frac{1}{2} \div 2\frac{1}{4}$	
	$\frac{15}{2} \div \frac{9}{4}$	
	$\frac{15}{2} \times \frac{4}{9}$	
	$\frac{5\,\cancel{15}}{2} \times \frac{4}{\cancel{9}\,3}$	
	$\frac{5}{1\,\cancel{2}} \times \frac{\cancel{4}\,2}{3}$	
	$\frac{10}{3}$	
	$3\frac{1}{3}$	

Trigonometric Identities

What Did I Do?	Finding the Solution(s)	Explain Reason(s) Why (or Ask a Question)
	$\cos^4\Theta - \sin^4\Theta = 1 - 2\sin^2\Theta$	
	$(\cos^2\Theta + \sin^2\Theta)\,(\cos^2\Theta - \sin^2\Theta) = 1 - 2\sin^2\Theta$	
	$(1)\,(\cos^2\Theta - \sin^2\Theta)$ $= 1 - 2\sin^2\Theta$	
	$\cos^2\Theta - \sin^2\Theta = 1 - 2\sin^2\Theta$	
	$(1 - \sin^2\Theta) - \sin^2\Theta = 1 - 2\sin^2\Theta$	
	$1 - 2\sin^2\Theta = 1 - 2\sin^2\Theta$	

MODELING WITH MANIPULATIVES

Strategy Note: For a discussion of the Modeling and Experimentation strategy—a strategic cousin of Modeling with Manipulatives—please see *Styles and Strategies for Teaching Middle School Mathematics* (Thomas & Brunsting, 2010, pp. 99–112) or *Styles and Strategies for Teaching High School Mathematics* (Thomas, Brunsting, & Warrick, 2010, pp. 110–119).

Purpose

Mathematics, as a discipline, is highly abstract. Consider this: 101 is not a prime number. It's not even a number—it's a series of symbols representing a numerical idea. Similarly, in geometry, points and lines can never truly be seen or defined. So symbols like • and ↔ are the best visual representations we have. Complex and simple mathematical ideas alike are represented by models all the time. Without these models, mathematics would be extremely difficult for students to comprehend. This is why manipulatives are invaluable resources for all teachers of mathematics. Whether an abacus or an app, manipulatives cut through abstraction and help students understand mathematical concepts in deeper and more personally meaningful ways.

Overview

Modeling with Manipulatives gives students a "feel" for mathematical concepts (often quite literally). Hands-on experiences with manipulatives can be fascinating "hooks" for introducing students to new concepts as well as powerful learning experiences that help students clarify and communicate their understanding. Modeling with Manipulatives lessons might engage students in the step-by-step process of transforming equations to understand equivalence, the simulation of real-world "chance" situations to understand probability, or the visual replication of a change in a linear equation variable to understand slope.

Building Common Core Thinking

Modeling with Manipulatives deepens students' conceptual understanding, providing a solid foundation in sense making, reasoning, and general problem solving. Modeling with Manipulatives supports the following Standards for Mathematical Practice (MP):

- ***(MP 1) Sense:*** using concrete objects to help conceptualize
- ***(MP 2) Reason:*** creating coherent representations and attending to the meaning of quantities
- ***(MP 3) Argument:*** constructing arguments using concrete referents such as objects, drawings, diagrams, and actions.
- ***(MP 4) Model:*** interpreting mathematical work in the light of a context
- ***(MP 5) Tools:*** using concrete models to solve problems

Steps

1. Identify an important mathematical concept on which you want students to focus.

2. Assemble the appropriate manipulatives and resources that students will need.
3. Create a student activity and determine whether students will work individually, with a partner, or in small groups. (*Note:* Having students work in pairs or small groups will generate helpful mathematical discussions and generally enhance the learning experience.)
4. Provide adequate instructional time so that student work and thinking are not rushed.
5. Have students reflect on their experience Modeling with Manipulatives and share how it has helped them explore and understand the concept.

Examples

Balancing Act

Provide students with small plastic cups (for water or medicine), several double-sided counters, paper, pencils, and rulers. Have students sketch a large "balancing scale" on paper—this scale should be more functional than aesthetic, as only two circular areas are needed for the cups and counters. (Alternatively, a balance scale template could be designed using a software program and printed out for students.) Instruct students that the placement of the cups and counters have meaning:

- upright cup = positive variable
- upside-down cup = negative variable
- light side of counter = positive value
- dark side of counter = negative value
- zero = equal number of light and dark counters

Model an equation to reinforce the concept for students. For example, $3x + 2 = 5$ would be modeled by three upright cups plus two light counters on the left side and five light counters on the right side. Once students are familiar with these manipulatives, they can solve any number of equations involving a single variable.

I KNOW WHAT I KNOW

Formative Assessment Connection: By quickly and thoroughly sharing what they know (including how and why their knowledge is accurate), students clarify their understanding of mathematics concepts for themselves and their teacher.

Purpose

"I don't know." Teachers of mathematics encounter this immediate, reflexive response from students far too often. Teachers recognize the crucial roles that accessing prior knowledge and reflection play in the learning process, but are often frustrated by their students' reluctance to engage in reflective thinking. I Know What I Know cuts through both abstraction and resistance, providing students with an easy-to-use format that encourages reflection, celebrates prior knowledge, and builds students' confidence in sharing *what* (sometimes *how* or *why*) they know about mathematical concepts.

Overview

I Know What I Know invites students to consider all that they know about a mathematical concept, procedure, or topic and confidently share their knowledge by completing a simple sequence of statements:

I know what I know about ______________________________ .

First, I know ______________________________ .

In addition, I know ______________________________ .

Finally, I know ______________________________ .

Now you know what I know about ______________________________ *!*

Alternatively, teachers may ask students to share how or why they know what they know to clarify their procedural knowledge or mathematical reasoning.

Building Common Core Thinking

I Know What I Know develops students' capacity for personal reflection, leading to deeper understanding and improved communication about their learning. I Know What I Know supports the following Standards for Mathematical Practice (MP):

- ***(MP 1) Sense:*** explaining correspondences between equations, descriptions, tables, and graphs or diagrams
- ***(MP 2) Reason:*** making sense of quantities and manipulating them
- ***(MP 6) Precision:*** formulating careful explanations

Steps

1. Select a topic or concept that is important for students to know well.
2. Decide whether students will consider and share

- *What* they know, in terms of factual information (e.g., definition, characteristics).
- *How* they know, in terms of procedural information (e.g., construction, transformation).
- *Why* they know, in terms of mathematical reasoning (e.g., proof, explanation, logic).

3. Model or review how to complete the I Know What I Know sequence.
4. Have students complete their I Know What I Know sequences individually.
5. Depending on concept and purpose, have students share their statements in pairs, small groups, or with the entire class.

Note: Consider extending student thinking by having pairs of students share what they know and ask each other, "Now, what do you know that I don't know?"

Examples

Visit the companion website for a blank "I Know What I Know Organizer."

Topic: Cubes (factual knowledge ⇒ "know about")

I know what I know about cubes.

- *First, I know* they make good dice.
- *In addition, I know* they have six sides.
- *Finally, I know* the volume of a cube is $l \times w \times h$ or just s^3.

Now you know what I know about cubes!

Topic: Finding Area of a Triangle (procedural knowledge ⇒ "how")

I know what I know about finding the area of a triangle.

- *First, I know how* to find a base of a triangle.
- *In addition, I know how* to find its altitude to that base.
- *Finally, I know how* to multiply $\frac{1}{2}$ times base times height.

Now you know what I know about finding the area of a triangle!

Topic: $x^2 + 5 > 0$ Must Always Be True (reasoning ⇒ "why")

I know what I know about why $x^2 + 5 > 0$ must be true.

- *First, I know why* $x^2 + 5 > 0$ because x^2 is always positive and the sum of two positive numbers is another positive number.
- *In addition, I know why* $x^2 + 5 > 0$, since x^2 is always positive, then $x^2 > -5$.
- *Finally, I know why* $x^2 + 5 > 0$ because $y = x^2 + 5$ represents a parabola with vertex (0, 5) and opening upward. It is always above the x-axis and therefore $x^2 + 5 > 0$.

Now you know what I know about why $x^2 + 5 > 0$ must be true!

What do you know that I don't know about why $x^2 + 5 > 0$ must be true?

TASK ROTATION

Strategy Note: For a discussion of the Task Rotation strategy, please see *Styles and Strategies for Teaching Middle School Mathematics* (Thomas & Brunsting, 2010, pp. 161–170) or *Styles and Strategies for Teaching High School Mathematics* (Thomas, Brunsting, & Warrick, 2010, pp. 169–180).

Formative Assessment Connection: Students' knowledge and understanding of a mathematics concept or topic is thoroughly assessed as they complete tasks using four styles of learning and thinking.

Technology Connection: Students can use many different technologies depending on task demands, from searching the web for information, to using calculators for analysis, to visualizing with dynamic graphing software.

Purpose

Engaging students. Differentiating instruction and assessments. Making sure students are thinking and working with mathematics concepts in various ways. Task Rotation helps teachers of mathematics address these goals through a simple yet powerful framework. The four-style quadrant of Task Rotation enables teachers to link four separate tasks together into a unified instructional and/or assessment piece focused around virtually any type of mathematics content. Because Task Rotations include tasks in all four styles, they naturally accommodate and challenge all learners of mathematics—students have the opportunity to work in their preferred styles and are encouraged to expand their mathematical reasoning by working in styles they tend to avoid. Students are engaged. Instruction and assessment are differentiated. And different types of mathematical thinking are explored in a compact form.

Overview

A good Task Rotation begins with the standards being addressed. For many teachers, this means the Common Core State Standards for Mathematics. By bringing together four tasks that require four distinct styles of thinking, Task Rotation enables teachers to support all eight Standards for Mathematical Practice. However, before designing the Task Rotation, teachers also should be familiar with the Standards for Mathematical Content for the course(s) they teach and identify the ones they and their students will focus on. After identifying the standards, the teacher is ready to think about the tasks that students are to complete. For each of the four styles, the teacher should ask, *What do I want students to be able to do in order to demonstrate that they can meet the standards?* (See Figure 5.11.)

For each style-based goal (for both teacher and students), the teacher should try to select a math tool that will help to achieve that goal. This is a surefire way to make sure that individual tasks require a different style of thinking and, together, that all four tasks support a variety of Standards for Mathematical Practice.

When designing a Task Rotation, it is always a good idea to create a "hook" or an engaging point of entry. A hook can be a quick activity, an informal discussion based on a provocative question related to the topic, or a narrative that orients

Figure 5.11 The Standards-Style Connection

Mastery What skills, procedures, and key terms do I want students to master?	**Interpersonal** How will students make personal connections or discover the social relevance of mathematics?
Understanding What core concepts, patterns, or principles do I want students to understand deeply?	**Self-Expressive** How will students explore, visualize, experiment, or apply new concepts and skills?

students to the mathematics featured in the Task Rotation. For example, here is one teacher's hook for her Task Rotation on linear equations and inequalities:

> *Think about the images you see every day in books, magazines, or movies, or on television, mobile apps, or computers. We live in a three-dimensional world, but these pictures on pages and screens are two-dimensional representations of objects, which consist only of points on a plane. So how can a two-dimensional picture represent a three-dimensional object? What are some other examples in which the impression of three dimensions is created in only two dimensions? Are all two-dimensional pictures designed to represent three-dimensional objects? Let's continue our investigation of linear equations and inequalities to help us get a better understanding of how they work mathematically, how they're graphed, and how they can be used to create images of common shapes and pictures.*

Building Common Core Thinking

Task Rotation builds students' mathematical reasoning, sense making, and problem solving through a series of style-based tasks. Task Rotation supports the following Standards for Mathematical Practice (MP):

- ***(MP 1) Sense:*** making sense of problems and persevering in solving them
- ***(MP 2) Reason:*** reasoning abstractly and quantitatively
- ***(MP 3) Argument:*** constructing viable arguments and critiquing the reasoning of others
- ***(MP 4) Model:*** modeling with mathematics
- ***(MP 5) Tools:*** using appropriate tools strategically
- ***(MP 6) Precision:*** attending to precision
- ***(MP 7) Structure:*** looking for and making use of structure
- ***(MP 8) Repetition:*** looking for and expressing regularity in repeated reasoning

Steps

1. Identify the standards (and practices) you want to address.
2. Connect your standards to the four learning styles. Consider these questions:
   - What skills, procedures, and key terms do I want students to master?

   - What core concepts, patterns, or principles do I want students to understand deeply?
   - How will students explore, visualize, experiment, or apply new concepts and skills?
   - How will students make personal connections or discover the social relevance of mathematics?

3. Design four tasks—one in each style—that will help you and your students address the standards and your four style-based goals. (*Note*: Drawing on tools from each of the four style-based chapters in this book is an efficient way to fill out your Task Rotation.)

4. Decide how you and your students will work through the Task Rotation:
   - Will you use the Task Rotation primarily for assessment (i.e., as a culminating task that asks students to demonstrate what they have already learned)?
   - Will you use the Task Rotation primarily for instruction (i.e., with completion of each task becoming a goal of instruction)? Do you need to provide instruction to develop the skills and content knowledge students will need to complete each task?
   - Will you ask students to complete all of the tasks, allow them to choose the tasks they wish to complete, or combine both choice and assignment?
   - Will you have students work in groups, independently, or both, depending on the demands of each task?

5. Design a hook that will pique students' interest, activate their prior knowledge, and help them connect that prior knowledge to the Task Rotation.

Examples

Task Rotation is truly a mighty vessel and the anchor of the mosaic or multi-style tools in this chapter. You can use Task Rotation in a variety of ways to differentiate instruction, assessment, or both. The examples that follow (Figures 5.12–5.16) offer a potpourri of mathematical Task Rotations. Before you start developing your own Task Rotations, review the examples on the following pages. Which of these Task Rotations are your favorites? Why? Which of them are closest to what you already do in your classroom? Which seem least like your classroom?

Figure 5.14 Task Rotation for *Polynomials*

Mastery

A polynomial and its terms are usually expressed in "simplified" form. Consider the two options for each expression below and identify the preferred or more correct option. Explain your choice.

Problem	Option I	Option II	Explanation
1	$1x$	x	
2	$9x^3$	$14x^3 - 5x^3$	
3	$\frac{6x^2y^5}{2x^2y}$	$3y^4$	
4	$5x^0$	5	
5	$2x^5 - x^4$	$-x^4 + 2x^5$	

(See Mathematical Conventions, p. 20)

Interpersonal

Think about what you confidently know about the arithmetic of polynomials. Complete the following sequence of statements:

I know what I know about polynomial arithmetic.

First, I know ________________________.

In addition, I know ___________________.

Finally, I know ________________________.

Now, you know what I know about polynomial arithmetic.

Share and compare what you know with a partner.

(See I Know What I Know, p. 162)

Understanding

The middle column below contains the solution to a problem. In the first column write what was done and in the third column explain why or state the property used.

What did I do?	Finding solution	Explaining why
Wrote original problem	$4x - 2x(x - 5)$	Starting point
	$4x + -2x(x + -5)$	
	$4x + -2x^2 + 10x$	
	$2x^2 + 4x + 10x$	
	$-2x^2 + 14x$	

(See Thinking Notes, p. 92)

Self-Expressive

Use Algebra Tiles to

A. Model and then solve the following:
   a. "x groups of (y + 3) things"
   b. "(x + 1) groups of (x + 1) things"

B. Create, model, and solve three original expressions of the following types:
   a. Product of a monomial and a binomial
   b. Product of a binomial and trinomial
   c. Perfect square binomial

(See Modeling With Manipulatives, p. 141)