Understanding Slope: A Key Concept in Algebra, Graphing, and Applied Rates

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Overview of the Lesson: Understanding Slope

**Common Core State Standards**
This lesson will introduce the concept of slope to students, and will help students to understand mathematical relationships between unit rates, average rates, graphs of lines, and steepness of lines. Teaching students about slope supports these Common Core State Standards:

**8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

**8.EE.6** Use similar triangles to explain why the slope m is the same between any two distinct point on a non-vertical line in the coordinate plane: derive the equation \( y = mx \) for a line through the origin and the equation \( y = mx + b \) for a line intercepting the vertical axis at \( b \).

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**Knowledge**

**What students should know:** In mathematics, *slope* is a number that describes the steepness of a line. Slope is a unit rate of change.

**Understanding**

**What students should understand:** Students will understand that the slope or steepness of the line represents the unit rate of change as it applies to the data that generates the graph.

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**Habits of Mind**

**What habits will students develop as a result of this lesson?** Students will apply Common Core Mathematical Practices to slope, rates, unit rates, linear equations, and linear graphs. Students will also see how the concept of slope can be connected to real world applications.

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**Skills**

**What will students be able to do?** Given a line or line segment drawn on a graphing grid, students will be able to determine its numerical slope. Given a numerical slope, students will be able to draw a line on the graphing grid, with the correct steepness that corresponds to the slope.

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**Key Vocabulary:** equation, linear equation, variable, graph, line, line segment, x-axis, y-axis, point, coordinates, plot, change, rate of change, unit rate of change, numerical slope
The Five Episodes of Teaching Understanding Slope

Preparing Students for New Learning

Paired Learner Model:
Students will be asked to recall memories and experiences with steep hills, as they relate to roads, ski slopes, bike rides, etc..

Students will be asked to intuitively select a number that represents the steepness of their hill.

Deepening and Reinforcing Learning

Modeling and Experimentation:
Students will experience a hands-on activity where they slide beads on graphing grids, generating lines with various slopes. Students will also learn how to determine the unit rate of change (slope) of a line given the coordinates of any two points on the line.

Game Competition: Students will review and reinforce their understanding and skills through Relay Races and friendly competition.

Delivering New Learning

Proceduralization and Paired-Learner Model:
Students will be introduced to slope, a number that represents the degree of steepness in a line. Students will be taught how to draw lines, with specified degrees of steepness, on a graphing grid. Students will coach each other as they practice.

Applying New Learning

Graduated Difficulty:
Students will experience connections between slope, unit rates of change, and real-world applications. Students will be encouraged to work at the level that feels right to them. Students will be encouraged to determine what they need to be ready to solve the higher level problems.

Reinforcing and Celebrating New Learning

Stand and Deliver:
This activity will provide students with opportunities to show and tell their new knowledge and understandings related to slope, and to earn congratulations for their efforts.
Preparing Students for Learning

Tool / Teaching Strategy: Paired Learner Model
Learning Style Addressed: Interpersonal

Hook: Recall a personal story related to slope. For example:
When I was a child, my sister and I walked 1/4 of a mile to the bus stop each day for school. Part of that walk included a very steep hill. One snowy day, we were walking down the treacherous hill and my sister slipped and fell. Her school books came out of her hands and slid all the way down the hill and into a storm sewer. When we arrived at school, we told our teacher about our ordeal. To this day, I remember that very steep hill.

Paired Learner Task:
Talk with a learning partner. Was there a time when you experienced a very steep hill? Maybe your experience involved a bicycle, skis, a car, etc.. Talk about your experience with your partner. Why does steepness matter? Be prepared to share your story with the class.

Connecting the Hook to the Math: (Teacher talking to students)
How steep was the hill in your story? Use a number to describe the steepness of your hill.

Teacher Tip: Survey five students. Write their names on the board, along with their numbers. Students will often choose numbers like 500, 1,000, or 75. The lesson that follows will teach students about slope, a number that mathematically describes the steepness of a line. Students will soon learn that many of their proposed numbers are way out of bounds.
Presenting and Acquiring New Learning

Tools / Teaching Strategies:
Proceduralization,
Paired Learner Model
Learning Style Addressed: Mastery

Slope and the Four-Step Process

Drawing a line with a specified slope \( k > 0 \) can easily be done using four simple steps.

**Step 1:** Place your pencil at the origin of the graphing grid and draw a point at the origin.

**Step 2:** Move the point of your pencil one unit to the right of the origin.

**Step 3:** Move the point of your pencil ‘north’, \( k \) units, equal to the slope. Draw a second point at this location.

**Step 4:** Draw a line that connects the two points. This line has slope \( k \).

Proceduralization Breakdown

- The teacher will model the four-step process.
- Each student will write and record the steps of the process.
- Paired learners A and B will take turns reading their steps to each other. As partner A reads aloud, partner B will listen and critique the work of A. Next, partner B will read his or her notes while partner B listens.
- The teacher will create two slope problems labeled A and B.
- Student A will solve problem A as B watches and provides coaching tips.
- Student B will solve problem B as A watches and provides coaching tips.
- The teacher will provide a third problem for all students to solve. Students will check their solutions with their learning partners and work together to resolve issues.

The Hook Revisited (Teacher talking to students)

Now that you know about slope, determine if the answers you submitted during the hook make sense or not. Report your answer and explain your reasoning.
Deepening and Reinforcing Learning: Part 1
Tools / Teaching Strategies:
Modeling and Experimentation
Learning Style Addressed: Understanding

Slope: A Hands-On Approach

- Provide each student with a full-page graphing grid and six graphing beads. Have students place the beads under the numbers 0, 1, 2, 3, 4, and 5.
- Ask students to multiply the numbers 0, 1, 2, 3, 4, and 5 by 1/2 and slide each number’s bead ‘north’ (above each number) a number of units equal to each product.
- Repeat the previous step for multipliers 1, 2, and 3.
- Regarding steepness, ask students to note similarities and differences in the geometric patterns of the beads.
- Students will discover that the multiplier m affects the steepness of each line.
- Tell students that the variable m in the equation $y = mx$ is the multiplier and is called the slope.
Deepening and Reinforcing Learning: Part 2

Tools / Teaching Strategies:
Modeling and Experimentation

Learning Style Addressed: Understanding

Slope: Rise / Run

Students will learn that the slope of a line can be calculated using the ratio of rise over run as it relates to any two points on a line.

Three linear graphs below show rise over run segments between two marked points on each line. Teach students how to divide the lengths of the rise and run segments to calculate the slope of each line. Stress the fact that the rise over run calculation of slope provides a value equal to the over one and up method learned earlier in the lesson.

Key Understanding: The rise over run calculation divides the total rise resulting from a run comprised of a number of unit lengths. The quotient is the rise value for one unit of run, or one unit rate of change.
Deepening and Reinforcing Learning: Part 3

Tools / Teaching Strategies:
Game Competition
Learning Style Addressed: All Four Learning Styles

**Game Competition:** At the end of the lesson, students will participate in Relay Races. The class will be divided into teams, and members form each team will compete in a fun and exciting game that provides a review and reinforcement of the concepts and skills learned in the lesson.

**Game Details:**
The class will be evenly divided into four or five teams A, B, C, D, E. Students can also be invited to create a personal name their team instead of using letters. The teacher will number each team’s members 1, 2, 3, 4, ... . One chair for each team will be placed at the front of the room facing the class. Associated with each chair will be a team label, a small white board, marker and cloth. Before the start of the Relay Races game, the teacher will model and review two or three problems that represent the types of problems the students will be solving during the game. To start the game each team’s player 1 will report to their team’s chair. The teacher will visit each chair, write a problem on each player’s white board, and instruct the students to place the board face down on their laps. At the teacher’s command GO, the students will flip their boards and solve their problem. Students should be given 1-2 minutes to solve their problem. Students who solve the problem correctly earn 1 point for their team. A student who solves the problem correctly and finishes first earns 2 points for their team. At the end of the time limit, players will take turns showing their solutions and explaining their reasoning to the class. The teacher will participate in the evaluation process help students to learn from any mistakes made. At the end of round 1, team points will be assigned and the players will return to their seats. Players 2 from each team will be invited to the front of the room to participate in round 2. As the rounds progress, students will demonstrate noticeable improvement in their understanding and problem-solving skills.
Applying Learning

Tools / Teaching Strategies: Decide Model / Graduated Difficulty
Learning Style Addressed: Interpersonal / Self Expressive

Choose and solve one of the three problems below.

1. In construction, the slope of a home’s roof is called the pitch of the roof. The pitch of a roof is calculated the same way the slope of a line is calculated. Calculate the pitch of the roof shown.

2. Andrew was a passenger on a jet that flew from Atlanta to Dallas. He was wondering, how steep is the flight path during take-off? Determine the slope of the flight path for the jet pictured to the right. Use your slope to determine the ground distance traveled as a jet rises to 10,000 feet.

3. A road sign warns drivers that the road ahead has a 12% grade. The grade of a road is a percent equal to the ratio of rise or fall relative to a specified horizontal length. Assume that a down-hill stretch of road has a 12% grade and spans a horizontal length of 1.5 miles. Calculate the slope of the road in feet and miles.

Teacher Note: Work with students to determine what makes each task progressively challenging. Students will need to know the knowledge, understanding, and skills they need to succeed at the higher levels.
Reflecting On and Celebrating Learning

Tools / Teaching Strategies:
Stand and Deliver
Game Competition

Learning Style Addressed: All Four Learning Styles

Stand and Deliver
Individual students will have the opportunity to stand and answer questions from the teacher. The questions will address the concepts and skills students learned in the lesson, and will span the four learning styles. The teacher will congratulate students as they demonstrate their new knowledge and understandings. The Stand and Deliver session will also provide the teacher with opportunities to uncover and correct any misconceptions some students might have.